

General rules

- You have 2 hours to complete the test. People with special facilities have 2h20 minutes in total.
- The exam is “closed book”, meaning that you can only make use of the material given to you.
- You are supposed to write the codes in Python language, but syntactic errors are allowed as far as the written algorithm can be well understood.
- For the loops, feel free to use for or whiles.
- The grade **for each Unit** will be computed as the number of obtained points, plus 1.
- Keep the names of the variables and functions as stated in the question.
- You are only allowed to use simple arithmetic operations (including modulo). Do not use any mathematical libraries or modules.
- **If you do not follow these instructions you will not receive any points in the respective question.**

Unit 1 (9 points)

A prime number is a natural number that it is only divisible by itself and 1. Write a function that receives N and returns a list with the first N prime numbers, just by iterating through all natural numbers and verifying it is prime. Note that 1 is not a prime number.

Unit 2 (9 points)

Assume that a matrix $A \in \mathbb{R}^{n \times n}$ can be written as:

$$A = \vec{a}\vec{a}^\top$$

with $\vec{a} \in \mathbb{R}^{n \times 1}$. Write a function that receives \vec{a} and a matrix $B \in \mathbb{R}^{n \times n}$ as one and two-dimensional arrays, respectively, and returns the matrix product AB as a two dimensional array. If the standard matrix-matrix multiplication has complexity $\mathcal{O}(n^{p+1})$, your algorithm should have complexity $\mathcal{O}(n^p)$. You need to explicitly compute p from the complexity analysis. In the code, for multiplying arrays, you need to use loops and not any pre-defined Python function.